XX JUBILEUSZOWA-KRAJOWA, VIII MIĘDZYNARODOWA KONFERENCJA NAUKOWO - TECHNICZNA
**„ZAOPATRZENIE W WODĘ, JAKOŚĆ I OCHRONA WÓD"**
XX-TH JUBILEE-NATIONAL, VIII-TH INTERNATIONAL SCIENTIFIC AND TECHNICAL CONFERENCE
**„WATER SUPPLY AND WATER QUALITY"**
POLAND 15-18 JUNE 2008

**Roman WICHOWSKI**

*Gdansk University of Technology*
*Faculty of Civil and Environmental Engineering*
*Gdańsk, Poland*

# WATER HAMMER ANALYSIS IN WATER SUPPLY NETWORKS

## ANALIZA UDERZENIA HYDRAULICZNEGO W SIECI WODOCIĄGOWEJ PIERŚCIENIOWEJ

*The paper presents method of analyzing of water hammer phenomenon in pipe networks. A mathematical model is presented by a set of partial differential equations of hyperbolic type, which have been transformed by the method of characteristics (MOC) into total differential equations, which are solved by the predictor-corrector method.*
*The main part of the paper is the application of appropriate equations in a difference form describing the unsteady flow phenomenon in complex pipeline systems, i.e. equations for nodes with branching or connecting pipelines. Such complex nodes can be found in various types of water supply systems, and also in district heating or industrial networks.*
*One calculation example is given related to the complex water-pipe network consisting of 17 real loops, 47 pipelines and 33 nodes, supplied by two independent sources. Water-hammer throughout the whole pipeline network were caused by closing the gate valve at midpoint of one selected pipe. The results of the numerical calculations are presented in a graphic form with respect to the final cross-sections of pipes.*

*W pracy przedstawiono metodę analizy zjawiska uderzenia hydraulicznego w sieci pierścieniowej. Model matematyczny zjawiska został opisany za pomocą układu dwóch równań różniczkowych cząstkowych pierwszego rzędu typu hiperbolicznego, które przekształcono za pomocą metody charakterystyk do układu dwóch równań różniczkowych zwyczajnych. Dla rozwiązania tych równań zastosowano metodę predykator-korektor, w której równania różniczkowe zwyczajne zostały zastąpione odpowiednimi równaniami różnicowymi.*
*Główna część pracy dotyczy zastosowania równań różnicowych, tzw. równań zgodności na odpowiednich charakterystykach, opisujących zjawisko przepływów nieustalonych, w złożonych układach przewodów z węzłami złożonymi. Węzły takie spotyka się w różnego rodzaju sieciach pierścieniowych, np. w sieci wodociągowej, ciepłowniczej oraz w sieciach przemysłowych.*
*W końcowej części pracy przedstawiono przykład obliczeniowy dla układu sieci pierścieniowej wodociągowej, składającej się z 17 pierścieni rzeczywistych, 47 przewodów i 33 węzłów, zasilanej z dwóch niezależnych źródeł, tj. zbiorników wyrównawczych początk-*

*kowych. Zjawisko uderzenia hydraulicznego zostało wywołane zamykaniem zasuwy wodociągowej, zamontowanej na środku jednego z przewodów. Wyniki obliczeń numerycznych zostały przedstawione w formie graficznej w odniesieniu do przekrojów końcowych wybranych dwóch przewodów, znajdujących się w pobliżu zasuwy.*

## 1. Introduction

Water-hammer in pipe networks result from an abrupt change in the flow velocity, by sudden changes in demand, abrupt closing or opening of liquid flow in a pipeline by means of various kinds of valves as well as uncontrolled pump starting or stopping. Water-hammer phenomenon is important in design, maintenance and operation of water distribution systems. It can cause high pressures and negative pressures, and the pipe can be damaged in the short term through over-pressures.

Although computer modeling tools for simulation of hydraulic transient flows have been widely used in simple pipeline systems, little is known about the behaviour of the transient flow in a complex pipe network systems. This phenomenon could be analyzed by a few numerical methods. One of them is the method of characteristics which could be used for very complex systems, e.g. distribution pipes in water supply systems and pipe networks. The main purpose of this paper is to put forward the method of analysis of transients flows in water supply networks. This subject-matter has been dealt with only by several scientific papers, mainly in the USA and Canada, cf. Streeter [8], [9], Karney and McInnis [5], McInnis and Karney [6], Wichowski [10, 11]. Particularly interesting is the paper of McInnis and Karney [6], where computerized transient-flow models have been used in the analysis of water-hammer events in topologically simple pipeline systems, and the performance of these models is well documented. The present paper analyzes the unsteady flow in a single straight steel pipe and in the complex pipe network. Results of calculations for a single pipe were verified with measurements performed on the experimental installation in hydraulic laboratory.

## 2. Solution of unsteady flows in simple pipes by the Method of Characteristics (MOC)

The mathematical model of the unsteady flows of compressible liquid in elastic pipes is expressed by the set of two partial differential equations of the first order of hyperbolic type, i.e. a momentum equation (1), and a relation of mass conservation (2), cf. [1], [2], [3], [9], [11]:

$$\frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + R_0 |Q|^m \operatorname{sgn} Q = 0 \tag{1}$$

$$\frac{\partial H}{\partial t}+\frac{a^2}{gA}\frac{\partial Q}{\partial x}=0 \tag{2}$$

where:

| | | |
|---|---|---|
| a | – | pressure wave speed; |
| A | – | cross-sectional area of pipe; |
| G | – | acceleration of gravity; |
| H | – | pressure head (piezometric head); |
| Q | – | fluid discharge; |
| $R_0$ | – | pipe resistance coefficient; |
| x | – | abscissa along the center line of the pipe; |
| t | – | time; |

The first equation is the momentum equation stating the dynamical equilibrium of the liquid particles in the cross-section of pipe, the second is the continuity equation derived from the mass conservation of the elastic fluid particles during their flow through an elastic pipe.

The paper presents a solution of the set of equations (1) and (2) by taking advantage of the characteristics method. At first it is necessary to transform them into suitable ordinary differential equations, referred to as compatibility equations on appropriate characteristics:

– compatibility equation on $C_+$ characteristic:

$$dH+\frac{a}{gA}dQ+R_0|Q|^m \operatorname{sgn} Q\, dx=0 \tag{3}$$

– compatibility equation on $C_-$ characteristic:

$$dH-\frac{a}{gA}dQ+R_0|Q|^m \operatorname{sgn} Q\, dx=0. \tag{4}$$

For the numerical calculation an iterative procedure, known as predictor-corrector method has been applied [2], [9], [10], [11]. Equations (3) and (4) are ordinary differential equations and they will be substituted by approximated difference equations, where finite increments $\Delta x$ and $\Delta t$ of independent variables x and t are used.

## 2.1. Equations for the internal point of the grid of characteristics

In the predictor-corrector method the corrected values of pressure head H(x,t), and the volume discharge of the liquid Q(x,t), or velocity v(x,t), are calculated on the basis of the mean resistance of an elementary pipeline section related to the discharge flow or velocity value of the proceeding and the following time steps. Making use of compatibility equations (3) and (4) we can write down the approximate difference equations for the internal points of the grid of characteristics (Fig. 1).

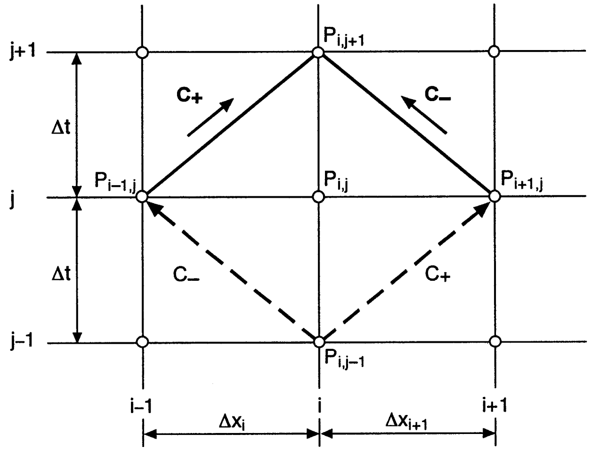


*Fig. 1. Elementary mesh of characteristics grid for internal points*

*Rys. 1. Elementarne oczko siatki charakterystyk dla punktów wewnętrznych*

## Predictor

$C_+$ compatibility equation:

$$H^p_{i,j+1} - H_{i-1,j} + Z_{i,j}\left(Q^p_{i,j+1} - Q_{i-1,j}\right) + R_{i,j}\left|Q_{i-1,j}\right|^m \operatorname{sgn} Q_{i-1,j} = 0 \tag{5}$$

$C_-$ compatibility equation:

$$H^p_{i,j+1} - H_{i+1,j} - Z_{i+1,j}\left(Q^p_{i,j+1} - Q_{i+1,j}\right) - R_{i+1,j}\left|Q_{i+1,j}\right|^m \operatorname{sgn} Q_{i+1,j} = 0 \tag{6}$$

## Corrector

$C_+$ compatibility equation:

$$\begin{aligned} &H_{i,j+1} - H_{i-1,j} + Z_{i,j}\left(Q_{i,j+1} - Q_{i-1,j}\right) + \\ &+ 0.5\,R_{i,j}\left(\left|Q_{i-1,j}\right|^m \operatorname{sgn} Q_{i-1,j} + \left|Q^p_{i,j+1}\right|^m \operatorname{sgn} Q^p_{i,j+1}\right) = 0 \end{aligned} \tag{7}$$

$C_-$ compatibility equation:

$$\begin{aligned} &H_{i,j+1} - H_{i+1,j} - Z_{i+1,j}\left(Q_{i,j+1} - Q_{i+1,j}\right) + \\ &- 0.5\,R_{i+1,j}\left(\left|Q_{i+1,j}\right|^m \operatorname{sgn} Q_{i+1,j} + \left|Q^p_{i,j+1}\right|^m \operatorname{sgn} Q^p_{i,j+1}\right) = 0 \end{aligned} \tag{8}$$

In the above equations R is the hydraulic resistance of the pipe or its section while value Z is the so-called characteristic impedance of pipeline expressed by the formula:

$$Z = \frac{a}{gA} \tag{9}$$

Having the approximated flow value at internal point $P_{i,j+1}$, it is now possible to find the corrected value of flow:

$$Q_{i,j+1} = Q^{P}_{i,j+1} + \Delta Q \tag{10}$$

where: $\Delta Q$ is the value of the correction, derived on the basis of Eqs. (5)-(8).

## 2.2. Equations for boundary points of the grid of characteristics

Examination of the grid of characteristics shows that the boundary points of the system being influencing the interior points after the first time step $\Delta t$. Therefore, in order to complete the solution to any desired time, it is necessary to include the boundary conditions. At either end of a single pipe only one of the compatibility equations is available. For the left end (Fig. 2a) equation (4) along the $C_-$ characteristic is valid, and for the right end (Fig. 2b) equation (3) along the $C_+$ characteristic is valid.
Left end (upstream end) (Fig. 2a):
**Predictor**

$C_-$ compatibility equation:

$$H^{p}_{0,j+1} - H_{1,j} - Z_{1,j}\,(Q^{p}_{0,j+1} - Q_{1,j}) - R_{1,j}\left|Q_{1,j}\right|^{m} \operatorname{sgn} Q_{1,j} = 0 \tag{11}$$

where: $H^{p}_{0,j+1} = H(x,t)$ or $Q^{p}_{0,j+1} = Q(x,t)$.

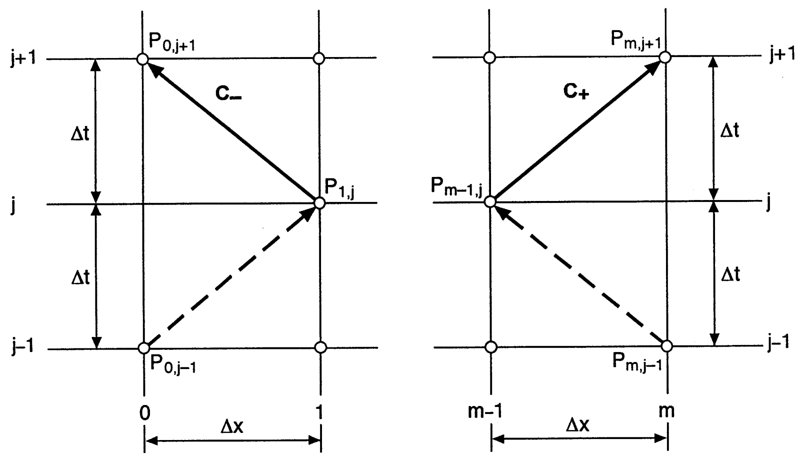


*Fig. 2. Elementary meshes of grid of characteristics for boundary points: a) left end; b) right end*

*Rys. 2. Elementarne oczka siatki charakterystyk dla punktów brzegowych: a) brzeg lewy, b) brzeg prawy*

**Corrector**

$C_-$ compatibility equation:

$$\begin{aligned}&H_{0,j+1}-H_{1,j}-Z_{1,j}\left(Q_{0,j+1}-Q_{1,j}\right)+\\&-0.5R_{1,j}\left(\left|Q_{1,j}\right|^m \operatorname{sgn} Q_{1,j}+\left|Q^p_{0,j+1}\right|^m \operatorname{sgn} Q^P_{0,j+1}\right)=0\end{aligned} \tag{12}$$

where: $H_{0,j+1}=H(x,t)$ or $Q_{0,j+1}=Q(x,t)$.

In the same way we can write appropriate equations for the right end of pipeline (downstream end) (Fig. 2b).

## 3. Analysis of unsteady flows in water-pipe networks

In the water supply systems, as well as in district heating networks and industrial complex pipelines systems are used generally branching pipes, i.e. forking and connecting pipes. Systems of this kind occur both in distributed and looped pipe networks, which consist of single transit pipelines, water mains, and distributing pipes. The junction points or nodes in systems of branching pipes are dealt with like appropriate internal boundary points. Thus, each junction point or node W (Fig. 3) is right end boundary with respect to an elementary section or sections of pipelines on its, or their, left side and simultaneously a left end boundary for an elementary section or sections of pipelines on the right side of that point.

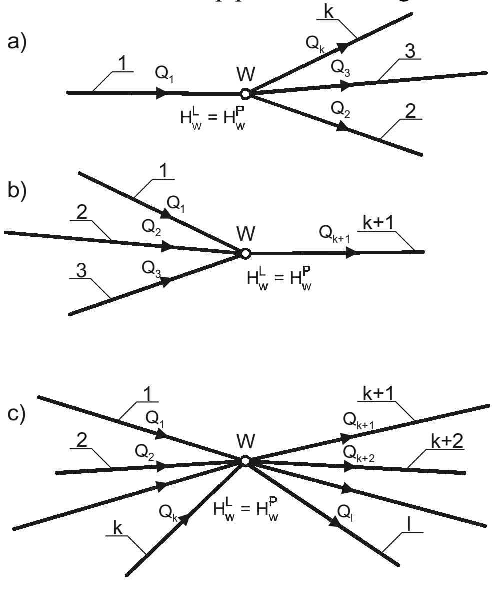


*Fig. 3. Examples of pipelines with different nodes: a) node with forking pipelines; b) node with connecting pipes; c) complex node*

*Rys. 3. Przykłady przewodów z różnymi węzłami: a) rozgałęzienie, b) połączenie, c) węzeł złożony*

In all cases the continuity equation, i.e. equilibrium condition of the sums of inflows to the junction and the sums of outflows from the junction, is utilized as the boundary condition:

$$\sum Q_{\text{inflows}} = \sum Q_{\text{outflows}} \tag{13}$$

It is additionally assumed that the pressure heads in the neighbourhood of point W are equal:

$$H_w^l = H_w^r \tag{14}$$

Supplementary equations are the compatibility equations with respect to appropriate characteristics depending on the case under consideration. Adequate equations for commonly applied branching junctions in complex pipeline systems will be presented below. For the junctions of forking pipelines (Fig. 3a) the following compatibility equations can be written:

$$\begin{aligned} H^{(1)}_{m1,j+1} - H^{(1)}_{m1-1,j} + Z^{(1)}_{m1,j}\left(Q^{(1)}_{m1,j+1} - Q^{(1)}_{m1-1,j}\right) + R^{(1)}_{m1,j}\left|Q^{(1)}_{m1-1,j}\right|^m \operatorname{sgn} Q^{(1)}_{m1-1,j} &= 0 \\ H^{(2)}_{0,j+1} - H^{(2)}_{1,j} - Z^{(2)}_{1,j}\left(Q^{(2)}_{0,j+1} - Q^{(2)}_{1,j}\right) - R^{(2)}_{1,j}\left|Q^{(2)}_{1,j}\right|^m \operatorname{sgn} Q^{(2)}_{1,j} &= 0 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\ H^{(k)}_{0,j+1} - H^{(k)}_{1,j} - Z^{(k)}_{1,j}\left(Q^{(k)}_{0,j+1} - Q^{(k)}_{1,j}\right) - R^{(k)}_{1,j}\left|Q^{(k)}_{1,j}\right|^m \operatorname{sgn} Q^{(k)}_{1,j} &= 0 \end{aligned} \tag{15}$$

where: $H^{(1)}_{m1,j+1} = H^{(2)}_{0,j+1} = H^{(3)}_{0,j+1} = \cdots = H^{(k)}_{0,j+1} = H_{j+1}$

and the continuity equation:

$$Q^{(1)}_{m1,j+1} = Q^{(2)}_{0,j+1} + Q^{(3)}_{0,j+1} + \cdots + Q^{(k)}_{0,j+1} \tag{16}$$

where superscripts (1), (2), ..., (k) denote the successive number of the pipeline.

Now let us the following denotations:

$$\begin{aligned} S^{(1)} &= -H^{(1)}_{m1-1,j} - Z^{(1)}_{m1,j}\, Q^{(1)}_{m1-1,j} + R^{(1)}_{m1,j}\left|Q^{(1)}_{m1-1,j}\right|^m \operatorname{sgn} Q^{(1)}_{m1-1,j} \\ S^{(2)} &= -H^{(2)}_{1,j} + Z^{(2)}_{1,j}\, Q^{(2)}_{1,j} - R^{(2)}_{1,j}\left|Q^{(2)}_{1,j}\right|^m \operatorname{sgn} Q^{(2)}_{1,j} \\ &\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ S^{(k)} &= -H^{(k)}_{1,j} + Z^{(k)}_{1,j}\, Q^{(k)}_{1,j} - R^{(k)}_{1,j}\left|Q^{(k)}_{1,j}\right|^m \operatorname{sgn} Q^{(k)}_{1,j} \end{aligned} \tag{17}$$

Making use of the denotations and equations (15) and (16), the following equations we obtained:

$$
\begin{aligned}
&H_{j+1} + Z^{(1)}_{m1,j}\, Q^{(1)}_{m1,j+1} + S^{(1)} = 0 \\
&H_{j+1} - Z^{(2)}_{1,j}\, Q^{(2)}_{0,j+1} + S^{(2)} = 0 \\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&H_{j+1} - Z^{(k)}_{1,j}\, Q^{(k)}_{0,j+1} + S^{(k)} = 0.
\end{aligned}
\tag{18}
$$

Taking into consideration the continuity equation (16), as well as the compatibility equation (15), the following set of (k+1) equations with (k+1) unknown values is derived:

$$
H_{j+1} = \frac{\dfrac{S^{(1)}}{Z^{(1)}_{m1,j}} + \dfrac{S^{(2)}}{Z^{(2)}_{1,j}} + \dfrac{S^{(3)}}{Z^{(3)}_{1,j}} + \cdots + \dfrac{S^{(k)}}{Z^{(k)}_{1,j}}}{\dfrac{1}{Z^{(1)}_{m1,j}} + \dfrac{1}{Z^{(2)}_{1,j}} + \dfrac{1}{Z^{(3)}_{1,j}} + \cdots + \dfrac{1}{Z^{(k)}_{1,j}}}
\tag{19}
$$

$$
\begin{aligned}
&Q^{(1)}_{j+1} = -\frac{H_{j+1} + S^{(1)}}{Z^{(1)}_{m1,j}} \\
&Q^{(2)}_{j+1} = \frac{H_{j+1} + S^{(2)}}{Z^{(2)}_{1,j}} \\
&\cdots\cdots\cdots\cdots\cdots\cdots \\
&Q^{(k)}_{j+1} = \frac{H_{j+1} + S^{(k)}}{Z^{(1)}_{1,j}}
\end{aligned}
\tag{20}
$$

In the same way we can write appropriate equations for junctions with connecting pipelines and for complex junctions where several pipes of positive and several pipes of negative flow direction meet in the node.

## 4. Distribution of transients pressure in water supply networks

Distribution of transients pressure in water supply looped networks is illustrated by a case related to the analysis of a middle-sized water supply network in unsteady conditions. The network consists of 17 real loops, 47 pipelines and 33 nodes, supplied by two independent sources, i.e. from the upper water reservoirs (Fig. 4).

The calculations of water supply network in steady state conditions were carried out using of EPANET 2 software elaborated by a group of research workers headed by Lewis Rossman from the U. S. Environmental Protection Agency, Cincinnati [7]. In the calculations of the looped network in steady state advantage was taken of input data as presented by the scheme of pipe network in Figure 4. The data include the number of the pipe, the length of the pipe section, and the nominal diameter, as well as information related to the nodes, as for instance, the node number, the external flow from the node, and the elevation of the node. It was assumed that the looped network would be made of cast iron flange pipes compatible with the Polish Standards. The calculations of the looped network in unsteady conditions were made using the WHAMMER computer program written in FORTRAN language based on the calculation algorithm presented by the author in the paper [10].

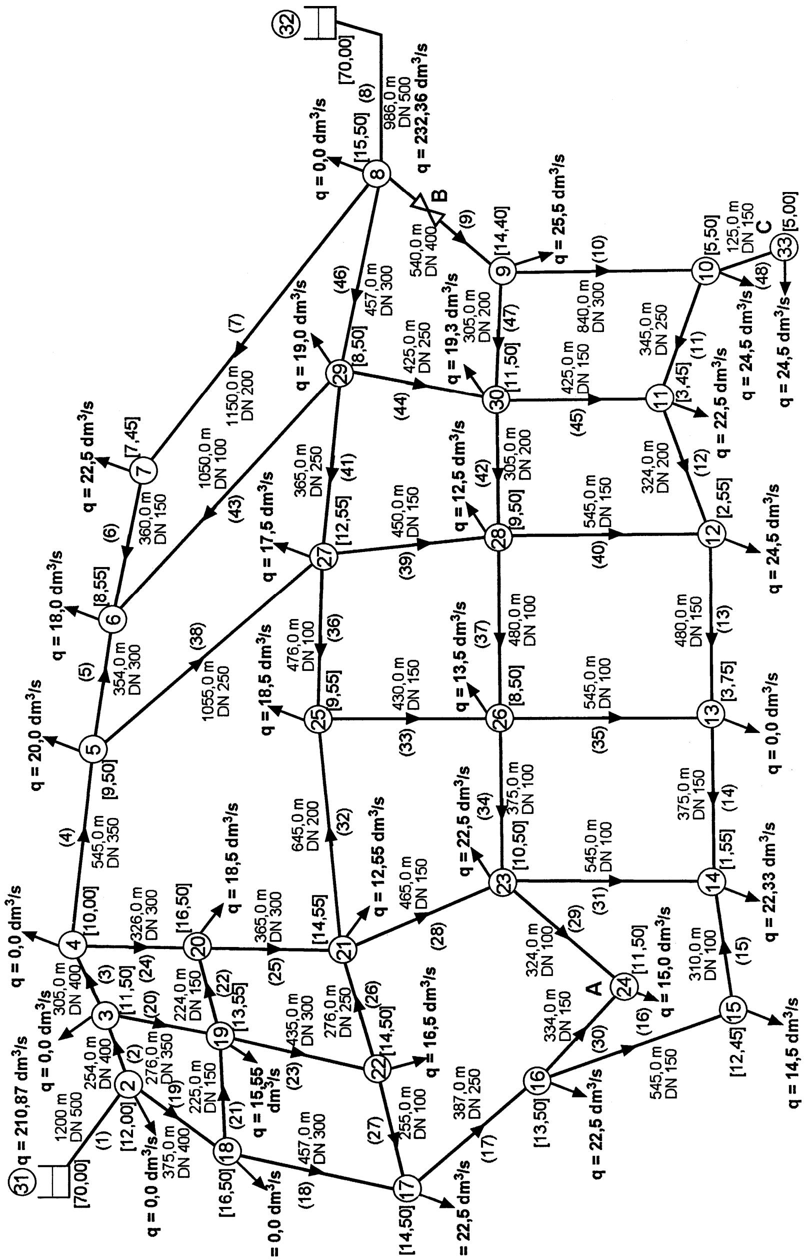

*Fig. 4. Schematic diagram of water pipe network for the Example*

*Rys. 4. Schemat sieci pierścieniowej do przykładu*

The unsteady flows in the network of Fig. 4 were caused by closing of the ball valve in time $T_c$ = 5 s, installed in the midpoint of pipe no. 9. The pipe of nominal diameter DN 400 and the total length L = 540 m is located in the vicinity of the upper reservoir supplying node no. 8 with water. The calculation results are presented in Figs. 5 to 6.

The analysis of the obtained results indicates that the largest pressure increments occur in the middle of the pipe before the ball valve on the side of node no. 8 (Fig. 4). The maximum pressure head will reach 207.4 m of $H_2O$, while the minimum one will be 4.6 m of $H_2O$ (Fig. 5). For this reason the higher pressure will be followed by pressure drop of 202.8 m, which is a magnitude twice bigger than the permissible working pressure for cast iron flange pipes according to Polish Standard PN-68/H-74101. The maximal value of the high pressure is in the permissible pressure range which in the case of cast iron flange pipes amounts to 2.5 MPa, that is about 250 m of $H_2O$.

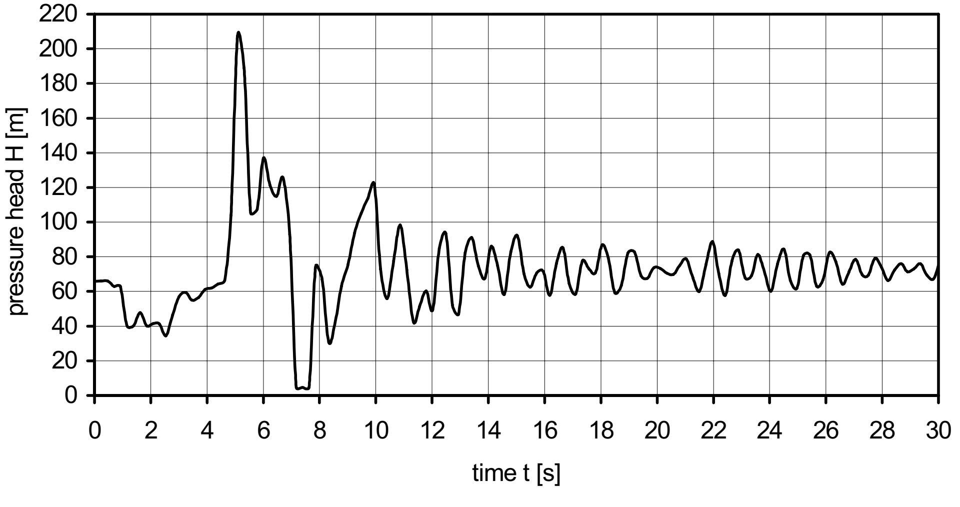


*Fig. 5. Pressure head oscillations at midpoint of pipeline no. 9 (node no. B)*

*Rys. 5. Wykres ciśnienia w punkcie środkowym przewodu nr 9 (węzeł nr B)*

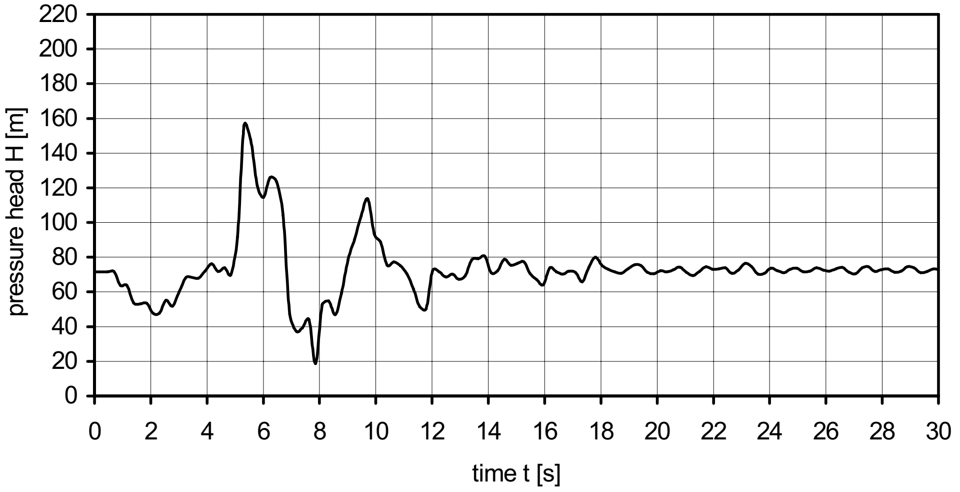


*Fig. 6. Pressure head oscillations at the final cross-section of pipeline no. 8 (node no. 8)*

*Rys. 6. Wykres ciśnienia w przekroju końcowym przewodu nr 8 (węzeł nr 8)*

The increment of pressure to the maximal value will occur in this case in pipe 9 after the valve has been completely closed, i.e. after 5 s. Further pressure oscillations are characterized by an evident reduction of the extreme values and in fact after 30 s the pressure heads will be close to the steady state value being approximately 70 m of $H_2O$. The maximum pressure head at the end of pipe no. 8 connecting reservoir 32 with the network is 156 m of $H_2O$, and the minimum value of the reduced pressure is 18.7 m of $H_2O$. However, the pressure oscillations around the steady state pressure amounting to approximately 60 m of $H_2O$ undergo a relatively fast damping and after 12 s have no significance taking the pipeline strength into consideration.

An examination of the extreme pressure heads in the remaining pipes outside pipe no. 9 where the flow disturbance occurred, has indicated that in this case we are dealing with remarkably large pressures in phases of higher values exceeding the permissible ones for cast iron pipes of ordinary wall thickness in 3 extra pipes, i.e. pipe no. 7 (node 7), pipe no. 46 (node 29), and in pipe no. 41 (node 27).

## 5. Summary and final conclusions

The main purpose of the paper was to present an adequate method for the analysis of transients flows in water pipe looped networks. To solve a set of partial differential equations describing the unsteady flow phenomenon in closed pipelines under pressure, there was applied *the method of characteristics (MOC)*, where the partial differential equations of hyperbolic type are transformed into ordinary differential equations, which have been solved by *the finite-difference method.*

In the numerical calculations advantage has been taken of the iterative process referred to as *the predictor-corrector method* which can be used in the cases when frictional effects are very important. In this method the corrected values of the unknows H(x,t) and Q(x,t) are calculated on the basis of the mean resistance of elementary pipe sections related to the flow value of the preceding and following time steps. Appropriate equations are used in the predictor-corrector method for internal nodes of the grid of characteristics, as well as for the boundary nodes. The main part of the paper is connected with presentation of suitable equations describing the unsteady flow phenomenon occurring in complex pipeline systems.

In the case of water pipe looped network, one computational example is given, in which the unsteady flows throughout the entire network were caused by a sudden closing the gate valve in the midpoint of pipeline no. 9. Numerical calculations were made with respect to the above specified system, and some results were presented in the paper in a graphic form, indicating the pressure heads under unsteady conditions in the selected final cross-sections.

The closing procedure of the gate valve installed at the midpoint of pipeline 9 with the time of closure $T_c$ = 5 s will be accompanied by minimal pressure head 4,6 m $H_2O$ and the maximal pressure head 207,4 m $H_2O$. The permissible pressure for cast iron flanged, or flared pipes of normal wall thickness is 1.0 MPa, i.e. approximately 100 m of $H_2O$.

The correctness of the applied procedure for analyzing the transients flows with regard to single pipes, has been verified by the use of our own unsteady-flow experiments. The investigations were carried out on a model situated at the Faculty of Environmental Engineering of the Warsaw University of Technology. The calculation results have indicated a good compatibility with the measurements not only with respect to our own investigations but also other research data. Due to limited space of the paper it was not possible to include the investigation results in the paper.

## Bibliography

[1] Chaudhry, M. H. Applied Hydraulic Transients, Van Nostrand Reinhold Company, New York, 1979.

[2] Evangelisti, G. Waterhammer analysis by the method of characteristics. L'Energia Elettrica, Nos. 10-12, 1969.

[3] Fox, J.A. Hydraulic Analysis of Unsteady Flow in Pipe Networks, The Macmillan Press Ltd, London and Basingstoke, 1977.

[4] Jeppson, R. W. Analysis of flow in pipe networks. Ann Arbor Science Publishers, Inc., Ann Arbor, Michigan, 1976.

[5] Karney, B.W., McInnis, D. Efficient calculation of transient flow in simple pipe networks. Journal of Hydraulic Engineering, 1992, 118 (7), 1014-1030.

[6] McInnis, D., Karney, B.W. Transients in distribution networks: field tests and demand models. Journal of Hydraulic Engineering, 1995, 121 (3), 218-231.

[7] Rossman, L.A. EPANET 2. Users Manual. Water Supply and Water Resources Division, National Risk Management Research Laboratory. Office of Research and Development, U.S. Environmental Protection Agency, Cincinnati, Ohio, September 2000.

[8] Streeter, V.L. Water-hammer analysis of distribution systems. Journal of the Hydraulics Division. Proc. of the ASCE, 1967, 93 (5).

[9] Streeter, V. L. Unsteady flow calculations by numerical methods. Proc. of the ASME. Journal of Basic Engineering, 1972, 94, series D, (2), 457÷466.

[10] Wichowski, R. Wybrane zagadnienia przepływów nieustalonych w sieci wodociągowej pierścieniowej (Selected problems of unsteady flows in water supply looped network). Wydawnictwo Politechniki Gdańskiej, seria monografie 27, Gdańsk, 2002, 1-210 (in Polish).

[11] Wichowski, R. Hydraulic Transients Analysis in Pipe Networks by the Method of Characteristics (MOC). Archives of Hydro-Engineering and Environmental Mechanics, 2006, 53(3), 3-26.

[12] Wylie, E.B., Streeter, V.L., Lisheng, S. Fluid transients in systems. Prentice Hall, Inc. A Simon & Schuster Company, New Jersey Englewood Cliffs, NJ 07632, 1993.